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FREE CONVECTION HEAT TRANSFER IN HORIZONTAL CHANNELS AND OVER A HORIZONTAL SURFACE

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Free convective heat transfer of a fluid in horizontal channels and above a horizontal surface is theoretically analyzed. The interaction of the fluid with the channel walls may be divided into two parts: formation of vortex flow cells, and heat transfer in the induced flow over the walls.

A great deal of experimental and theoretical work has been devoted to investigating heat transfer processes in free convection, but as yet no theory exists to determine the degree of heat transfer to horizontal channels and surfaces. Heat transfer conditions are analyzed in references [1, 2, 3] near the point of instability only. The free motion of a fluid at large Ra values is examined in references [4, 5], but the authors determine only average temperature profiles, and the results obtained are not confirmed by the experimental data of [6, 7]. Known analytical solutions [8, 9], based on boundary layer theory, are applicable only for vertical heating surfaces in a fluid of infinite extent.

An account is given below of a theory of free convection heat transfer in horizontal channels and over horizontal surfaces based on the following experimental facts:

1. The flow of a fluid in horizontal channels in the range of Ra numbers from 1700 to 45 000 has a pronounced cellular structure [10, 11].

2. When the channel width increases to infinity, the cellular structure of the fluid is maintained in the immediate vicinity of the heat transfer surface [6].

3. In the intermediate range, when Ra > 45 000, the data of [6] show that the cellular structure of the fluid is maintained on the lower heat transfer surface.

Possible fluid structures are shown schematically in Fig. 1.

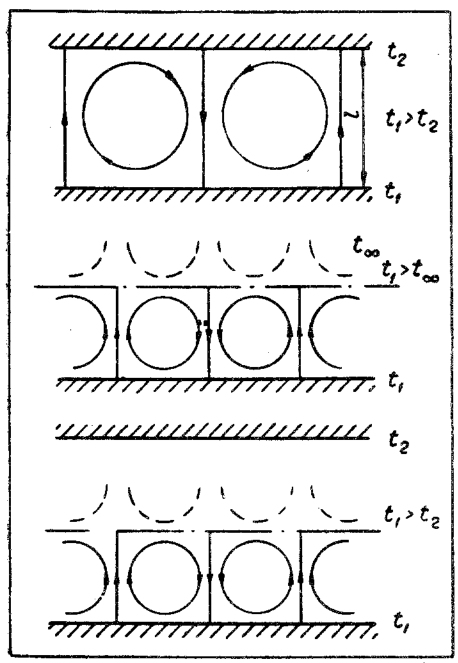

Fig. 1. Fluid flow patterns.

We shall consider that the intensity of heat transfer under the conditions considered here depends mainly on the velocity of the fluid in the cellular layer existing near the solid surface. The problem may then be reduced to the calculation of the thermal boundary layer developing on the solid surface as a result of the interaction of the paired vortices forming the cellular structure. The circulation velocity of the fluid in the cells is determined primarily by bouyancy forces, which do not have a notable influence on the law of the thermal boundary layer at the wall. Thus, the problem of free convection under the conditions considered here may be divided into two parts:

1. Determination of the intensity of circulation of the fluid in the cells.

2. Calculation of the thermal boundary layer in the induced fluid flow close to the heat transfer surface.

Let us examine the flow of a fluid with constant physical properties in a plane channel in the region Ra < 45 000. Figure 2 shows the flow patterns under these conditions. The vertical velocity component in the vortex cylinder in the horizontal section at the moment of onset of instability is determined, according to [12], by

$$v_y = v(y)\cos(\pi/l)\,x. \qquad (1)$$

Let us assume that a similar velocity distribution holds even for developed vortex flow at $y = l/2$:

$$v_1 = -v_0\cos(\pi/l)\,x. \qquad (2)$$

We find the velocity distribution in the other sections from the conditions of potential flow in the cell (Fig. 2b).

The general solution for the stream function has the form

$$\psi = \sum_{n=1}^{\infty} A_n \operatorname{sh} \frac{\pi n y}{l} \sin \frac{\pi n x}{l}. \tag{3}$$

We find the coefficients of the series from the following relation:

$$A_n \operatorname{sh} \frac{\pi n l}{2l} = \frac{2}{l} \int_0^l f(x) \sin \frac{\pi n x}{l}\, dx, \tag{4}$$

where

$$f(x) = \psi_1 = \frac{v_0 l}{\pi} \sin \frac{\pi x}{l}.$$

Therefore,

$$A_1 = 0.435\, v_0 l/\pi; \quad A_2 = A_3 = A_4 = 0.$$

The horizontal velocity near the surface of the plate is

$$v_x = (\partial \psi/\partial y)_{y=0} = 0.435\, v_0 \sin \pi x/l. \tag{5}$$

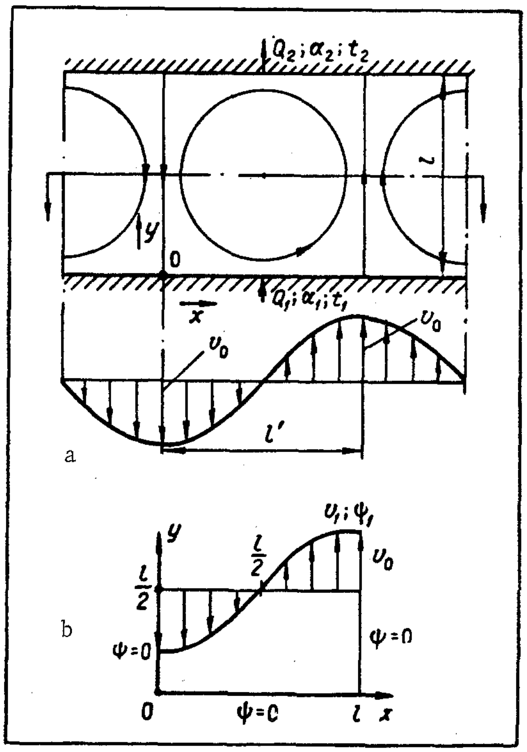

Fig. 2. Scheme for calculating potential flow flow of fluid in cell and velocity distribution in the horizontal section (a) and other sections (b).

Thus, the first part of our problem has been solved correct to within the as yet unknown value of the maximum velocity v_0 in the vortex. To determine this velocity, and also to solve the second part of the problem, we have the following set of equations:

momentum equation for the boundary layer that develops on the plate due to the induced fluid flow of velocity v_x [13]:

$$\frac{d\,\mathrm{Re}^{**}}{dX} + \left(1 + \frac{\delta^*}{\delta^{**}}\right) \frac{\mathrm{Re}^{**}}{v_x} \frac{dv_x}{dX} = \mathrm{Re}_l \frac{c_f}{2}; \tag{6}$$

law of friction for a laminar boundary layer [13]:

$$c_f/2 = 0.22/\mathrm{Re}^{**}; \tag{7}$$

equation of the thermal boundary for the case Δt = const [13]:

$$d\,\mathrm{Re}_T^{**}/dX = \mathrm{Re}_l\, \mathrm{St}; \tag{8}$$

heat transfer relation for a laminar boundary layer [13]:

$$\mathrm{St} = 0.22/\mathrm{Re}_T^{**}\, \mathrm{Pr}^{1.2}; \tag{9}$$

heat balance equation (Fig. 2a):

$$Q_1 = -Q_2; \tag{10}$$

equation of equilibrium of forces acting on the liquid circulating in the cells.

From (6) and (7), with the boundary condition $(\mathrm{Re}^{**})_{X=0} = 0$ and $\delta^*/\delta^{**} = 2.5$, we have

$$\mathrm{Re}^{**} = 0.663 \left(\frac{l c v_0}{\nu \pi} f(\pi X) \right)^{1/2}, \tag{11}$$

where $c = 0.435$ and

$$f(\pi X) = (\sin \pi X)^{-7} \left[0.546 \times \left(\frac{\pi X}{2} - \frac{1}{4} \sin \pi X \right) - \right.$$

$$\left. - 0.125 (\sin \pi X)^7 \cos \pi X - 0.146 (\sin \pi X)^5 \cos \pi X - 0.182 (\sin \pi X)^3 \cos \pi X \right].$$

From (11) and (7) we obtain

$$\tau_W = Z(\pi X)\rho c^{3/2} v_0^{3/2} \sqrt{\nu/l}, \tag{12}$$

where

$$Z(\pi X) = 0.586(\sin \pi X)^2/[f(\pi X)]^{1/2}.$$

The values of function $Z(\pi x)$ are respectively equal to 0; 0.565; 1.110; 1.230; 1.140; 0.895; 0.537; 0.201; 0.029; 0.001; 0 when X = 0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1.

For the mean value of the sheer stress at the wall due to one vortex cylinder we have

$$\overline{\tau}_W = 0.574\rho c^{3/2} v_0^{3/2} \sqrt{\nu/l}. \tag{13}$$

The friction force at the wall is

$$W = \overline{\tau}_W l = 0.574\rho c^{3/2} v_0^{3/2} \sqrt{\nu l}. \tag{14}$$

From (8) and (9), with the boundary condition $(\mathrm{Re}_T^{**})_{x=0} = 0$, we have:

$$\mathrm{Re}_T^{**} = \left(\frac{0.44 c v_0 l}{\nu \mathrm{Pr}^{1.2}\pi}(1 - \cos \pi X)\right)^{1/2}. \tag{15}$$

Substituting in (9), we obtain a formula for the average heat transfer coefficient

$$\overline{\alpha} = 0.53\lambda \mathrm{Pr}^{0.4} \sqrt{c v_0 / l \nu}. \tag{16}$$

The quantity of heat transmitted by the lower wall to the fluid is

$$Q_1 = 0.53\lambda l \mathrm{Pr}^{0.4}(t_1 - t_0)\sqrt{c v_0 / l \nu}. \tag{17}$$

The quantity of heat transmitted by the fluid to the upper wall is

$$Q_2 = 0.53[t_2 - (t_0 + \Delta t')]\lambda l \mathrm{Pr}^{0.4} \sqrt{c v_0 / l \nu}, \tag{18}$$

where

$$\Delta t' = Q(c_p \gamma v_{av} l/2)^{-1}, \tag{19}$$

$$v_{av} = (2/l)\int_0^{l/2} v_0 \cos(\pi x/l)\,dx = 0.637 v_0.$$

Therefore,

$$t_0 = (t_1 + t_2 - \Delta t')/2. \tag{20}$$

The buoyancy force acting on the fluid circulating in a cell is

$$W_1 = \Delta t' \beta \gamma l^2/2, \tag{21}$$

where $l^2/2$ is the volume in which the buoyancy flow takes place.

A closed set of equations results, which may be solved by the method of successive approximations. Since $\Delta t' \ll \ll t_1 + t_2$, we shall take $\Delta t' = 0$ as the first approximation in (20). Equating the friction forces at the wall ($W = 2\overline{\tau} l$) and the buoyancy forces, we have

$$v_0 = (0.364 \Delta t \beta g l / c \mathrm{Pr}^{0.6})^{1/2}, \tag{22}$$

where

$$\Delta t = t_1 - t_2.$$

We determine a value of Q from (17) and of $\Delta t'$ from (19), and obtain a new value of t_0 from (20). From this value of t_0 we finally obtain an expression for Nu for free convection of a fluid in narrow horizontal channels:

$$\mathrm{Nu} = 0.17\,(1 + 0.58/\mathrm{Ra}^{0.25}\,\mathrm{Pr}^{0.2})\,\mathrm{Ra}^{0.25}. \tag{23}$$

The problem of free convection from a horizontal plate in a fluid of infinite extent may be similarly solved. In this case a thin vortex layer forms near the heat transfer surface, depending on the ratio of viscous to buoyancy forces. From dimensional considerations the thickness of the layer is given by

$$l = (a\nu\,\mathrm{Ra}_{cr}/\Delta t\,\beta g)^{1/3}, \tag{24}$$

where

$$\Delta t = t_1 - t_\infty.$$

According to [6], $\mathrm{Ra}_{cr} = 10^4 - 2.5 \cdot 10^4$. Solving the set of equations (6)-(10) with the conditions $t_0 = t_\infty$; $\alpha_1 \leq \alpha_2$; $W = W_1$; $W = \overline{\tau_w l}$, we have:

$$v_0 = (1.45\,\Delta t\,\beta\,gl/c\mathrm{Pr}^{0.6})^{1/2} \tag{25}$$

and

$$\bar{\alpha} = 0.47\,\lambda\,\mathrm{Ra}_{cr}^{-0.0835}\,(\Delta t\,\beta\,g/\nu a)^{1/3}. \tag{26}$$

When $\mathrm{Ra}_{cr} = 2.5 \cdot 10^4$, we obtain

$$\mathrm{Nu} = 0.2\,\mathrm{Ra}^{1/3}. \tag{27}$$

On the basis of parametric correlation of experimental data on heat transfer in free convection from a horizontal plate the following formula [14] has been proposed:

$$\mathrm{Nu} = 0.18\,\mathrm{Ra}^{1/3}. \tag{28}$$

Thus, quite satisfactory agreement is obtained between the theoretical formula and experimental data.

On the basis of this theory we can obtain, in the first approximation, an analytical expression for the heat transfer in free convection in a horizontal channel in the region Ra > 45 000. In this case vortex flow similar to that which occurs on a horizontal plate is observed at the lower wall. The picture of the flow near the upper wall is not sufficiently clear, but if we make the assumption that $\alpha_1 \approx \alpha_2$, and carry out a calculation similar to that above, we obtain the formula

$$\mathrm{Nu} = 0.08\,\mathrm{Ra}^{1/3}. \tag{29}$$

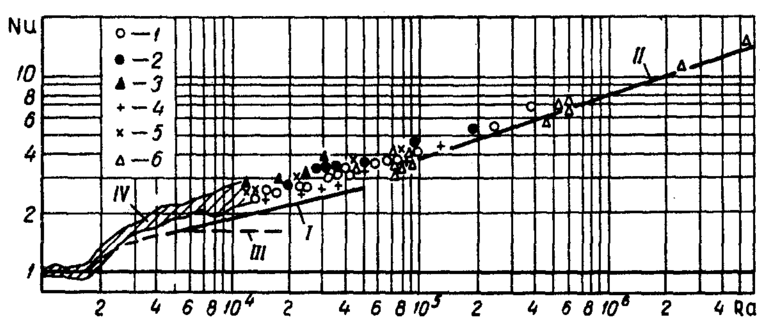

Fig. 3. Comparison of theory and experiment. Experiments of [15]: 1) water; 2, 3) silicone oil AK-3, AK-350 (Pr ≈ 4000); 4) heptane; 5) ethylene glycol. Experiments of [16]: 6) water; I) according to (23) (Pr = 0.73); II) according to (29); III) according to Nakagawa's theory [3]; IV) region of experimental points of [15].

In Fig. 3 existing experimental data are compared with values of the heat transfer in free convection in horizontal channels calculated according to our theory. It can be seen from the graph that there is qualitative agreement in the region Ra < 45 000, and good quantitative agreement in the region Ra > 45 000.

NOTATION

v_0—maximum value of velocity in middle section of cellular layer; ψ—stream function; l—thickness of cellular layer; v_x—horizontal velocity component near plate surface; $Re^{**} = v_x \delta^{**}/\nu$—variable Reynolds number based on momentum thickness; ν—kinematic viscosity; δ^*—displacement thickness; δ^{**}—momentum thickness; δ_T^{**}—energy loss thickness; $Re_l = v_x l/\nu$—Reynolds number, based on velocity v_x and characteristic dimension; $X = x/l$—relative distance; c_f—friction coefficient; St—Stanton number; $Re_T^{**} = v_x \delta_T^{**}/\nu$—Reynolds number, based on energy loss thickness; ρ—density; γ—specific weight; λ—thermal conductivity; β—coefficient of volume expansion; g—acceleration due to gravity; α—thermal diffusivity; Ra_{cr}—Rayleigh number based on thickness of cellular layer and $\Delta t = t_1 - t_\infty$; t_0—average temperature in descending flow; $\Delta t'$—difference of average temperatures in the descending and ascending flows; v_1—velocity at $y = l/2$; v_y—vertical velocity component. Subscripts: "cr"—critical, "w"—wall, 2—vertical component; x—horizontal component.

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